

Melosh rotation: source of the proton's missing spin

Bo-Qiang Ma

Center of Theoretical Physics, CCAST (World Laboratory), Beijing, People's Republic of China

and

Institute of High Energy Physics, Academia Sinica, PO Box 918 (4), Beijing 100039, People's Republic of China

Abstract. It is shown that the observed small value of the integrated spin structure function for protons could be naturally understood within the naive quark model by considering the effect from Melosh rotation. The key to this problem lies in the fact that the deep inelastic process probes the light-cone quarks rather than the instant-form quarks, and that the spin of the proton is the sum of the Melosh rotated light-cone spin of the individual quarks rather than simply the sum of the light-cone spin of the quarks directly.

The spin content of the proton has received extensive attention from the particle physics community recently. The reason for this is that the European Muon Collaboration (EMC) found [1] from their polarized muon proton data a much smaller value of the integrated spin structure function for protons compared with that from the Ellis-Jaffe sum rules [2]. This small integrated spin structure function combined with the Bjorken sum rule [3] was interpreted as the evidence that a very small fraction of the proton's spin is provided by the spin of its quarks. This conclusion, if true, is of course startling because it is in clear contradiction with the previous theoretical expectations [4]. Hence, many papers have been devoted to this problem and many complicated models for the proton's missing spin have been proposed. In this letter, we indicate, however, that the small value of the proton's integrated spin structure function could be naturally understood in the naive quark model (NQM) by considering the effect of the Melosh rotation [5, 6].

The key to this problem lies in two very simple but in practice often mistakenly treated or ignored facts. The first is that the deep inelastic lepton scattering process is a probe of the light-cone (or current) quarks rather than the instant-form (or constituent) quarks [7, 8, 9]. The second is that in light-front dynamics the spin of the proton is not simply the sum of the spin of the individual quarks but the sum of the Melosh rotated spin of the light-cone quarks [10, 11]. The theoretical bases for the two facts can be traced back to the old work of Dirac's relativistic Hamiltonian dynamics [12], Weinberg's

infinite momentum technique [13], and Wigner's spin state rotation [14]. The first fact leads naturally to the conclusion that the quark's spin measured in deep inelastic lepton scattering is the light-cone spin rather than the instant-form spin. Taken in conjunction with the second fact, we can conclude that there is no need to require that the sum of the quark's spin measured in deep inelastic process be equal to the proton's spin.

In the following, we simply present an intuitive model to evaluate the effect from Melosh rotation. We start from the conventional instant-form (T) $_{\text{NQM}}$ SU(6) proton wavefunction

$$|p_T^\uparrow\rangle = (2u_T^\uparrow u_T^\uparrow d_T^\downarrow - u_T^\uparrow u_T^\downarrow d_T^\uparrow - u_T^\downarrow u_T^\uparrow d_T^\uparrow)/\sqrt{6} \text{ (+cyclic permutation)} \quad (1)$$

one finds $\Delta u_T = \frac{4}{3}$, $\Delta d_T = -\frac{1}{3}$ and $\Delta s_T = 0$. If the deep inelastic process is a probe of the instant-form quarks, we expect, respectively, the integrated spin structure function for protons

$$\int g_1^p dx = \frac{1}{2}(\frac{4}{9}\Delta u + \frac{1}{9}\Delta d) = \frac{5}{18} = 0.278 \quad (2)$$

and that for neutrons

$$\int g_1^n dx = \frac{1}{2}(\frac{1}{9}\Delta u + \frac{4}{9}\Delta d) = 0 \quad (3)$$

together with the proton's spin sum rule

$$(\Delta S_Z^T)_{u+d+s} = \frac{1}{2}(\Delta u_T + \Delta d_T) = \frac{1}{2} \quad (4)$$

which means that the proton's full spin is carried by its valence quarks.

The instant-form (T) quark states q_T^s and the front-form (F) quark states q_F^s are related by the Melosh rotation [5, 6, 10, 11]

$$q_F^s = \sum_{s'} M_{s's}(R) q_T^{s'} \quad (5)$$

with the Melosh rotation operator defined by

$$R = (m + k_0 + k_3 + i\varepsilon_{ij3}\sigma_i k_j)/[2(k_0 + k_3)(m + k_0)]^{1/2} \quad (6)$$

in specifying q_F^s and q_T^s by the two-component Pauli spinors. From (5), we get, inversely,

$$q_T^\uparrow = w[(k^+ + m)q_F^\uparrow - k^R q_F^\downarrow] \quad q_T^\downarrow = w[(k^+ + m)q_F^\downarrow + k^L q_F^\uparrow] \quad (7)$$

in which $w = [2k^+(m + k_0)]^{-1/2}$, $k^{R,L} = k_1 \pm ik_2$, $k^+ = k_0 + k_3$ and $k_0 = (m^2 + \mathbf{k}^2)^{1/2}$. We see from (7) that the light-cone spin carried by an instant-form quark should be its instant-form spin multiplied by a factor

$$M_q = [(k^+ + m)^2 - \mathbf{k}_\perp^2]/[2k^+(m + k_0)]. \quad (8)$$

Therefore we can identify

$$\Delta q_F = \langle M_q \rangle \Delta q_T \quad (9)$$

in which M_q is the contribution from the Melosh rotation.

We simply assume that the quark momentum-space wavefunction of the proton is described by the harmonic oscillator wavefunction

$$\Psi(\mathbf{k}) = \pi^{-3/4} \alpha^{-3/2} \exp(-\mathbf{k}^2/2\alpha^2). \quad (10)$$

We know from previous work [15] that this wavefunction is good in describing the static properties of hadrons at low momentum scale in adopting the harmonic scale $\alpha \approx 330$ MeV and the quark mass $m \approx 330$ MeV. At high momentum scale one may expect that m becomes smaller or α becomes larger [15]. Hence the expectation value of M_q may be evaluated by

$$\langle M_q \rangle = \int d^3\mathbf{k} M_q |\Psi(\mathbf{k})|^2. \quad (11)$$

As deep inelastic scattering is a probe of the light-cone quarks, we should use Δq_F rather than Δq_T in equations (2)-(3) to calculate the integrated spin structure functions for protons and neutrons. Assuming $M_u = M_d$ and adopting m, α to be 134 MeV, 330 MeV or 330 MeV, 815 MeV respectively, we obtain

$$\int g_1^p dx = \frac{1}{2} \left(\frac{4}{9} \Delta u_F + \frac{1}{9} \Delta d_F \right) = \frac{5}{18} = 0.126 \quad (12)$$

and

$$\int g_1^n dx = \frac{1}{2} \left(\frac{1}{9} \Delta u_F + \frac{4}{9} \Delta d_F \right) = 0 \quad (13)$$

together with

$$(\Delta S_z^F)_{u+d} = \frac{1}{2} (\Delta u_F + \Delta d_F) = 0.227 \quad (14)$$

which means that the sum of the light-cone spin of the valence quarks is only 45.4% of the proton's spin. From (12) we see that the above intuitive picture could naturally explain the small EMC data of the proton's integrated spin structure function with reasonable parameters.

One can easily find that the above results are quantitatively inconsistent with the Bjorken sum rule. This comes from the adoption of the $_{\text{NQM}}$ SU(6) proton wavefunction and the assumption $M_u = M_d$. Actually the proton's instant-form wavefunction should be

$$|p_T\rangle = a_0 |uud\rangle_T + a_1 |uudq\bar{q}\rangle_T + a_2 |uudg\rangle_T + \dots \quad (15)$$

in which the high Fock state contributions could change Δu_T and Δd_T from the values $\frac{4}{3}$ and $-\frac{1}{3}$. $\langle M_u \rangle$ and $\langle M_d \rangle$ may also be different since there are two u valence quarks and one d valence quark in the proton. Bearing the above consideration in mind, we start from the most recent EMC data

$$\int g_1^p dx = \frac{1}{2} \left(\frac{4}{9} \Delta u_F + \frac{1}{9} \Delta d_F \right) = \frac{5}{18} = 0.126 \quad (16)$$

and the Bjorken sum rule

$$\int (g_1^p - g_1^n) dx = \frac{1}{6} (\Delta u_F - \Delta d_F) = \frac{1}{6} g_A/g_V \quad (17)$$

with $g_A/g_V = 1.259$ determined from neutron β decay [16] to evaluate the values of $\Delta u_T, \Delta d_T, \langle M_u \rangle$ and $\langle M_d \rangle$. In order to simplify the discussion, we neglect the possible

effects from the sea or gluon polarization[‡] and from the quark or gluon orbital angular momentum. From (16) and (17), we obtain

$$\Delta u_F = \langle M_u \rangle \Delta u_T = 0.705 \quad \Delta d_F = \langle M_d \rangle \Delta d_T = -0.554 \quad (18)$$

and the sum of the light-cone spin of the valence quarks

$$(\Delta S_z^F)_{u+d} = \frac{1}{2}(\Delta u_F + \Delta d_F) = 0.076 \quad (19)$$

which is very small. We know that $\Delta u_{T,F}$, $\Delta d_{T,F}$, $\langle M_u \rangle$ and $\langle M_d \rangle$ should meet the general requirements

$$-2 \leq \Delta u_{T,F} \leq 2 \quad -1 \leq \Delta d_{T,F} \leq 1 \quad 0 \leq \langle M_{u,d} \rangle \leq 1 \quad (20)$$

and the spin sum rule

$$\frac{1}{2}(\Delta u_T + \Delta d_T) = \frac{1}{2} \quad (21)$$

The combination of equations (18), (20) and (21) leads to the constraints

$$1.554 \leq \Delta u_T \leq 2 \quad 0.352 \leq \langle M_u \rangle \leq 0.454 \quad -1 \leq \Delta d_T \leq -0.554 \quad (22)$$

$$0.554 \leq \langle M_d \rangle \leq 1.$$

Therefore in order to satisfy both the EMC data and the Bjorken sum rule simultaneously, it is necessary that $\langle M_u \rangle \neq \langle M_d \rangle$, $\Delta u_T \neq \frac{4}{3}$, and $\Delta d_T \neq -\frac{1}{3}$; i.e. the proton's instant-form valence quark distribution should be different from that of the $_{\text{NQM}}$ SU(6) wavefunction, and the u quark and the d quark should have different momentum-space wavefunctions in the proton. We are also interested to see that the Melosh rotation is also an important source for the depletion of g_A/g_V relative to the value $\frac{5}{3}$ expected from the SU(6) naive quark model. This is a significant source in comparison with other sources such as the effect from the quark transverse momenta [18] and the effect due to 'small' components in the quark's Dirac spinors in the bag model [19] or in quark-confining potentials [20].

In summary, we present in this letter a very simple model in which the EMC results of the proton's integrated spin structure function could be naturally explained within the naive quark model by considering the effect from Melosh rotation. This model does not necessarily invalidate the Bjorken sum rule if we impose some constraints on the Fock state wavefunction of the proton. This work is based on two very simple but profound facts which have sound bases both theoretically and experimentally. Though the quantitative results in this letter may be changed by the complicated effects from the sea and gluon polarizations and by contributions from the orbital angular momentum, or by the anomalous gluon contributions via the U(1) axial anomaly, the effect from Melosh rotation should be of fundamental importance in the spin content of hadrons and therefore should not be ignored. We think the effect revealed in this letter should have also manifested itself in a number of high energy processes, and therefore requires further theoretical and experimental works.

[‡] Close [17] indicated recently that the magnitude of the (strange) sea polarization is likely to be significantly nearer to zero than is being assumed in much of the current literature.

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